Developing Key Competences by Mathematics Education

Lifelong Learning Programme

Concepts for in-service teacher education
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Concepts for In-Service Mathematics Teacher Education: Examples from Europe

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1 Introduction[1]

Teachers are considered to play a central role when addressing professional development programmes: "Teachers are necessarily at the center of reform, for they must carry out the demands of high standards in the classroom" (Garet, Porter, Desimone, Birman, & Yoon, 2001, p. 916). Ingvarson, Meiers, and Beavis (2005) sum up: "Professional development for teachers is now recognised as a vital component of policies to enhance the quality of teaching and learning in our schools. Consequently, there is increased interest in research that identifies features of effective professional learning" (p. 2).

In the past 20 years, newly emerging challenges for the teaching profession have resulted in an increased demand for corresponding (new) professional competences and an adequate framework (Posch, Rauch & Mayr 2009).

Until the 1990s, the role of teachers was mostly limited to offering pre-determined contents in a manner that is as clear and illustrative as possible, to ensuring discipline and assessing performance. This "static" culture of school-based teaching and learning has come under pressure in recent years, both with regard to student/teacher interaction and how classroom work is defined in substantive terms. Increasingly, the norms applicable at school are at variance with the wealth of extramural experiences children and adolescents gather. As a consequence, there are efforts to further develop and instil dynamic momentum into the static culture of teaching and learning.

The conditions of work and employment have changed: Graduates are increasingly expected to have multi-functional skills. They should be able to perform conceptual, planning and supervisory tasks. Higher demands are being placed in terms of a self-reliant design of work processes. And ultimately – due to the growing complexity of work settings – team work has gained increasing importance. Requirements such as these have created a demand for what has been called "dynamic" skills, for self-reliance, independent management and use of knowledge, and a sense of responsibility.

The socialisation which children and adolescents have experienced before they enrol at school has altered significantly in the past 30 years. Families tend to have fewer children, whilst the individual child plays a more central role. The relationship of parents and children has shifted to one that revolves around partnership. What is allowed and what is forbidden is more and more the outcome of a negotiating process between children and parents. It is with this background of experiences that children and adolescents come to school, projecting it onto this institution. As the prevailing work culture at school rests on a social model in which children and adolescents accept authoritarian decisions of adults, the resultant potential for conflict is inevitably huge.

Against this backdrop, teachers are facing challenges on a professional and human scale which are novel in many aspects. KeyCoMath aims at changes of pupils' learning. In particular, more active, exploratory, self-regulated, autonomous, communicative and collaborative learning is intended. Research on this kind of learning usually focuses on the students' level. Questions like these are raised: How can students' learning be well defined? How can we describe and explain students' progress and difficulties? However, these changes of pupils' learning are based on changes of teaching. Teachers develop expertise to arrange learning environments in order for pupils to work in the intended way and to develop key competences. Research on the teachers' level

[1] This introduction is based on Rauch, Zehetmeier & Erlacher (2014) and Zehetmeier, Rauch & Schuster (in preparation).

includes questions like: How can teachers be supported in activities dealing with this kind of learning? What do we learn from research when supporting teachers in implementing these learning activities? Teachers are considered to play a central role in planning, implementing, and researching professional development programmes.
This section provides some insights and examples, how various countries deal(t) with these issues. In particular, examples from Austria, Bulgaria, Germany and South Tyrol are provided.

2 Austria: The IMST Project[2]

Whereas in the last decades of the twentieth century many countries launched reform initiatives in mathematics and science instruction, or concerning teaching and teacher education in general, similar systemic steps in Austria did not happen. This led to a big gap between intended and implemented instruction, both in schools and at teacher education institutions. Although the promotion of students' understanding, problem solving, independent learning, etc. and the use of manifold forms of instruction and didactic approaches in mathematics and science instruction were regarded as important, teacher-centred instruction and application of routines dominated. Retrospectively, it seems clear that the educational system needed an external impulse, and this appeared in the late 1990s in the form of TIMSS 1995 and later PISA.
The initial impulse for the IMST project[3] in Austria came from the TIMSS achievement study in 1995. Whereas the results concerning the primary and the middle school were rather promising, the results of the Austrian high school students (grades 9 to 12 or 13), in particular with regard to the TIMSS advanced mathematics and physics achievement test, turned out to be disappointing (and evoked discussions on educational practice, research and policy, influenced by critical reports in the media). The ranking lists showed Austria as the last (advanced mathematics) and the last but one (advanced physics) of 16 nations (see e.g. Mullis et al. 1998, pp. 129, 189). This and other indicators showed that the teaching of mathematics and science in Austria needed a shift from "transmission" to "inquiry".

As in many other countries (see e.g. Prenzel and Ostermeier 2006), the responsible ministry reacted to the situation. In Austria, a national initiative with the aim to foster mathematics and science education was launched in 1998: the IMST project. Since then, this initiative has undergone several adaptions and is still running.
IMST was implemented in three steps:

1. The task of the *IMST research project* (1998–1999) was to analyse the situation of upper secondary mathematics and science teaching in Austria and to work out suggestions for its further development. This research identified a complex picture of diverse problematic influences on the status and quality of mathematics and science teaching: For example, mathematics education and related research was seen as poorly anchored at Austrian teacher education institutions. Subject experts dominated university teacher education, while other teacher education institutions showed a lack of research in mathematics education. Also, the overall structure showed a fragmented educational system consisting of lone fighters with a high level of (individual) autonomy and action, but little evidence of reflection and networking (Krainer, 2003; see summarized in Pegg & Krainer, 2008).

2. The *IMST² development project* (2000–2004) focused on the upper secondary level in response to the problems and findings described. In addition, it elaborated a proposal for a strategy plan for the ministry, aiming at improving the inquiry-based learning (IBL) of STEM in secondary schools. The two major tasks of IMST² were (a) the initiation, promotion, dissemination, networking and analysis of innovations in schools (and to some extent also in teacher education at university); and (b) recommendations for a support system for the quality development of mathematics, science and technology teaching. In order to take systemic steps to overcome the "fragmented educational system", a "learning system" (Krainer, 2005) approach was taken. It adopted enhanced reflection and networking as the basic intervention strategy to initiate and promote innovations at schools. Besides stressing the dimensions of reflection and networking, "innovation" and "working with teams" were two additional features. Teachers and schools defined

[2] This example is based on Krainer & Zehetmeier (2013) and Zehetmeier & Krainer (2013).
[3] IMST = originally, Innovations in Mathematics and Science Teaching (1998-1999); later, Innovations in Mathematics, Science, and Technology Teaching (2000-2009); since 2010 - motivated by adding German studies as one more subject - Innovations Make Schools Top.

their own starting point for innovations and were individually supported by researchers and project facilitators. The IMST^2 intervention built on teachers' strengths and aimed to make their work visible (e.g., by publishing teachers' reports on the website). Thus, teachers and schools retained ownership of their innovations. Another important feature of IMST^2 was the emphasis on supporting teams of teachers from a school.

3. The *IMST3 support system* (in four stages 2004–2006, 2007–2009, 2010–2012, 2013–2015, a fifth stage 2016–2019 is in preparation) started to implement parts of the strategy plan, among other ways by continuously broadening the focus to all school levels and to the kindergarten, and also to the subject German language (due to the poor results in PISA). The overall goal of IMST is to establish a *culture of innovation* and thus to strengthen the teaching of mathematics, information technology, natural sciences, technology, and related subjects in Austrian schools (see e.g. Krainer et al. 2009). Here, culture of innovation means starting from teachers' strengths, understanding teachers and schools as owners of their innovations, and regarding innovations as continuous processes that lead to a natural further development of practice, as opposed to singular events that replace an ineffective practice (for more details see e.g. Altrichter and Posch 1996; Krainer 2003).

For the future, the ministry expressed its intention to continue IMST. The overall goal is setting up and strengthening a culture of innovations in schools and classrooms, and anchoring this culture within the Austrian educational system.

3 Bulgaria: How can Teachers be Supported in Inquiry-Based Mathematics Education with Focus on Key Competences

3.1 Background

To create a class culture in which the teachers and the students could work as a research team using the ICT in support of the inquiry-based learning has been the goal of a long-term research in Bulgaria dating from the early 80's. The first attempts are related with the Research Group on Education (RGE) – having carried out an educational experiment launched by the Bulgarian Academy of Sciences and the Ministry of Education in 1979 (Sendov 1987, Sendov, Filimonov, Dicheva 1987; Sendova 2011). It comprised 29 pilot schools (2 % of the Bulgarian K-12 schools) and its main goal was to develop a new curriculum designed to make the use of computers one of its natural components. The guiding principles of RGE were learning by doing, guided discovery, and integrated school subjects. The RGE experiment ran for 12 years. Spreading the positive experience of RGE on a broader scale at the time turned out to be very difficult for various reasons – both economic and political. However, even with these isolated experiments the lessons learned were valuable – *the learners' and teachers' creativity alike can be enhanced when provided with appropriate environments.*

At a university level new courses were introduced, reflecting the need to prepare teachers working in the style of project based learning and guided discovery learning promoted in the RGE schools, e.g. at the Faculty of Mathematics and Informatics at Sofia University the curriculum for future mathematics teachers was enriched by the course *Teaching Mathematics in Laboratory Type Environment* (Nikolova, Sendova, 1995). After years of studying and reproducing very sophisticated mathematical facts these teachers-to-be experienced situations in which they could say: "Look at my construction!", "Can you prove my theorem?" (Denchev, Kovatcheva, Sendova 2012). Such a spirit of discovery was expected to be transferred later on to their students. Still, a negative tendency (noticed not only in Bulgaria but also in the most of Eastern Europe) was the slow decline of the educational system in the 90's and the early 2000's (Denchev, Kovatcheva, Sendova 2012).

3.2 New Life for Inquiry-Based Learning

With the advent of powerful ICT and specially designed educational software for mathematics explorations a way was opened for inquiry-based learning (IBL) in a number of European countries. Mathematics was an especially important domain of IBL thanks to the development and the dissemination of dynamic learning environments in which various experiments with mathematical objects could be performed leading to the formulation and verification of hypotheses of the learners themselves. Activities of this kind were the focal point of the involvement of the Bulgarian *KeyCoMath* team members in previous European projects, including *InnoMathEd, Fibonacci, DynaMath, Math2Earth, Mascil* and *Scientix* (Kenderov 2010; Kenderov, Sendova, Chehlarova, 2012).

Our work with in-service teachers is based on the understanding that for the teachers to be motivated they should experience the same intellectual pleasure we expect their students to undergo. The inquiry-based mathematics education (IBME) is promoted by our team at two levels – nationally and locally, in major regional centres. On a national level the promotion instruments are workshops, seminars and special sections of the national conferences organised by the Union of Bulgarian Mathematicians. On a local level IBME is promoted and supported by multiple training and presentation sessions organised in fifteen Bulgarian regions with the help of local boards (Chehlarova, Sendova 2012; Sendova, Chehlarova 2012).

3.3 Activities and Resources in Support of Inquiry Based Mathematics Education

The activities of the Bulgarian KeyCoMath research team in terms of organising events and developing educational resources embrace the four levels of the IBL:

Level 1 - Confirmation Inquiry, in which students confirm a principle through an activity whereby the results are known in advance;

Level 2 - Structured inquiry, in which students investigate a teacher-presented question through a prescribed procedure;

Level 3 - Guided inquiry, in which students investigate a teacher-presented question through a procedure they designed/selected themselves;

Level 4 - Open inquiry, in which students investigate a question they have formulated themselves through a procedure they designed themselves (Banchi, Bell 2008; Sendova 2014).

Here is a short description of such activities and resources:

PD courses (from 2 to 128 hours)

These courses are being organized by the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences (IMI-BAS) in the framework of European projects (InnoMathEd, Fibonacci, Mascil, KeyCoMath and Scientix), as well as by sections of the Union of Bulgarian Mathematicians (UBM), by the Ministry of Education and Science, by publishing houses for educational literature, and by PD centers. The main goal of the courses is in harmony with the most recent educational strategies for updating the math and science education in the EC countries: the development of key competences by implementing the inquiry based learning in integration with the world of work. These PD courses are based on a team work (of the lecturers and the participants alike) and implement educational models adaptable to various school settings. The crucial part of the courses is for the participants to experience different stages and levels of IBL with emphasis on key competences.

Each summer in the years 2014 and 2015 the KeyCoMath team has offered training courses for IBL for teachers in Bulgaria who are not involved directly in the project but want to learn and use the methods of IBL. The teachers work on pedagogical problems related with:

- reformulating math problems in IBL style so as to enhance the development of specific key competences;
- formulating their own math problems reflecting real-life situations, not solvable with the current math knowledge of the students but allowing for explorations by means of dynamic geometry models leading to a good enough approximation of the solution;
- studying and proposing methods for tackling problems which are unstructured, or whose solutions are insufficient or redundant;
- solving “traditional problems” with “non-traditional” data, for which the use of a computing device is necessary;
- applying game-design thinking so as to engage better the students in the problem solving;
- formulating more relevant evaluation criteria for the students’ achievements;
- project-based work with presentation of the results;
- assessment of learning resources in terms of formation and development of IBL skills and key competences (Kenderov, Sendova, Chehlarova 2014; Chehlarova, Sendova 2014).

The courses have two phases. In the first phase (3 days in the beginning of the summer) the teachers become acquainted with the dynamic geometry software GeoGebra as well as with plenty of examples of how to use dynamic scenarios in the IBL style. The participants of the courses are assigned projects as a “home-work”. In the second “follow-up” phase (with duration 1-2 days and conducted at the end of the summer) the participants present the advancement in the work on their project in front of the other participants in the course.

PD events (seminars and workshops) in the frames of conferences

The key feature of these events is that the teachers play an active role and act as partners in a research team – they share their good practices in oral or poster presentations (sometimes jointly with their students), work in groups on specific tasks and present their ideas to the rest of the participants. Typical examples include the *Scientix National Conference* within the National seminar *Inquiry Based Mathematics Education*, the *Dynamic Mathematics in Education* conference (Fig. 1), the seminars within the Spring conferences of UBM**,** the regional conferences organized by UBM sections, the International UNESCO workshop *QED* (Chehlarova 2012; Sendova 2015).

Fig 1. The Scientix National Conference demonstrated good practices of teachers implementing IBL with emphasis on specific key competences

The inquiry based learning, its connection with the world of work, good practices and problems directed at the development of specific key competences, have been the focus of our work with in-service teachers:

Using specific learning scenarios in support of IBL

A good repository of such resources is the *Virtual School Mathematics Laboratory* (Fig. 2, http://www.math.bas.bg/omi/cabinet/) being developed by IMI-BAS (Chehlarova, Gachev, Kenderov, Sendova, 2014), which contains over 800 scenarios with dynamic files transparent for the users. The way teachers are encouraged to use these resources is to stimulate students to behave like working mathematicians: to make experiments, to look for patterns, to make conjectures, to verify them experimentally, to apply "what-if" strategies so as to modify/generalize the problem, and even to use them as a preparation for a rigorous proof. To do this without leaving their comfort zone, the teachers enter the role of their students and experience the same type of activities during our courses, and when working on their own. They first use the dynamic files supporting the scenarios as a ground for explorations. The next step for them is to propose appropriate modification of the files for similar problems, or to use them as a model for creating one of their own from scratch. Thus, it would be quite natural for them to tell the students: "I don't know the answer, but I hope to find it together with you, thanks to YOUR efforts, to our joint efforts…"

Fig. 2. Virtual School Mathematics Laboratory: dynamic files for the open problem on finding the locus of a regular m-gon inscribed in a regular n-gon

Building and developing competences necessary for the students to participate in new types of mathematics contests

Examples are "Mathematics with a computer" and "Theme of the month" (Fig. 3, see Kenderov, Chehlarova 2014; Chehlarova, Kenderov 2015).

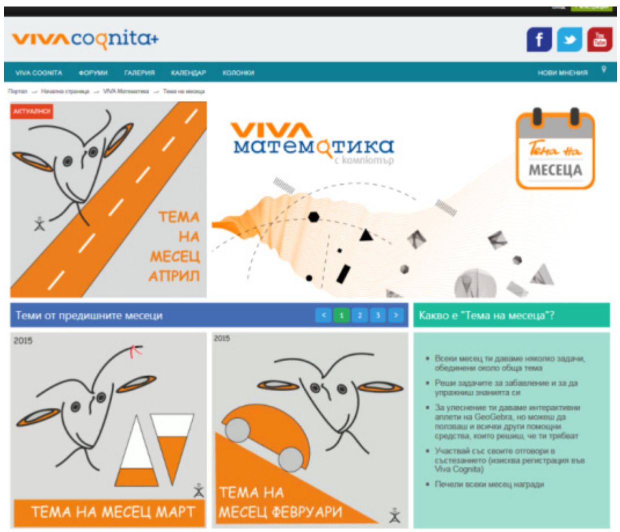

Fig 3. "Theme of the month": an invitation for a long-term activity on a chain of math problems modeling real-life situations

Mathematics performances

Events raise the awareness of the general public about the role of mathematics for enhancing children's scientific curiosity and endeavor to learn. The examples include: Performance at the History Museum in Stara Zagora, organized by the UBM section in the town, performances during the Researchers' Nights (2011-2015), Science festivals (in Italy, Romania, Greece). It is important to note that the teachers act as multipliers of the IBL ideas during these events as well – they participate with their students, and occasionally lead the performance.

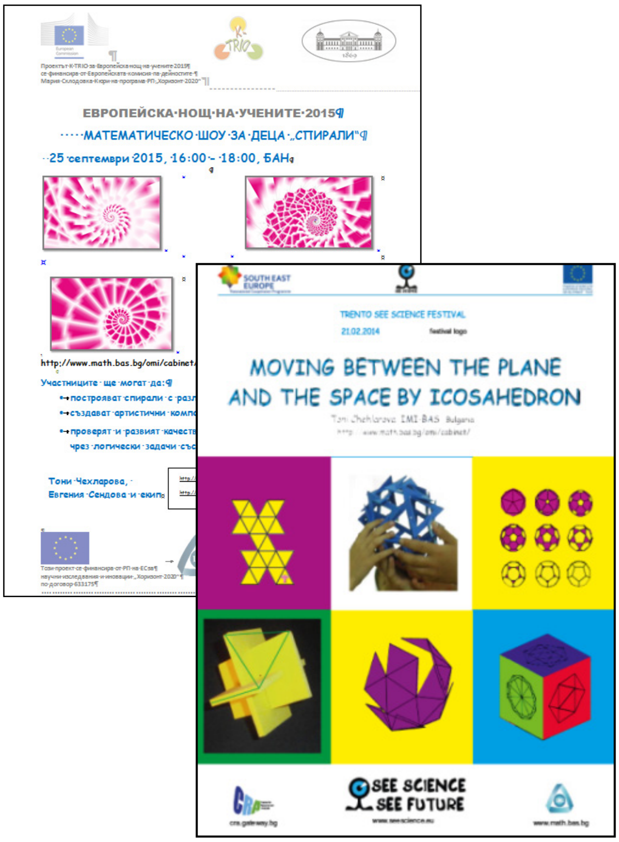

Fig. 4. Posters for math performances within Science Fairs and Researchers' nights

Individual work with teachers

It includes support for the development of a lesson, educational materials, mathematical festivals, course projects, peer reviews, and preparation of a pedagogical experiment.

Activities in support of the Open Inquiry

The fourth level of IBL (the Open Inquiry) has been promoted to reach teachers and students from all over the country with potential to do research in mathematics and informatics. This has been done in the frames of the High-School Students' Institute of Mathematics and Informatics (HSSI) where secondary school students work (under the supervision of a mentor) on their own projects, focused on the study of a given problem from Mathematics, Informatics and/or IT (Kenderov, Mushkarov, Parakozova 2015). The students deliver their results and findings, in written and oral form, in front of a jury and peers at two sessions – in January and April. A three week Summer School is organized for the involved teachers and the best achieving students. During the Summer Research Schools special seminars for teachers are organized. During these seminars experienced teachers share their good practices in mentoring students with high potential in doing research, and professional researchers in mathematics and informatics together with PhD students (HSSI alumni) deliver lectures on contemporary topics in these fields suggesting possible topics for future research projects.

3.4 The Main Achievements

A community of teachers who implement and spread the inquiry based learning of mathematics and informatics has been created. They participate in pedagogical experiments not only as a reality-proof of researchers but as members of a research team. These teachers implement, modify and develop from scratch educational resources in support of IBL, share their good practices at seminars, national and international conferences and in professional journals. Some of them organize public events at a school and regional level for popularizing the inquiry based mathematics education. Teachers are also key figures in organizing the new mathematics contests *Mathematics with a computer* and *Theme of the month*, in making them known to a broader audience.

The further goal is to reach faster and more effectively larger groups of other teachers and school students in acquiring the IBL approach with focus on the development of key competences.

The activities of the High School Institute of Mathematics and Informatics and the teachers results are encouraging. For its 15 years of existence it has proved that research potential of the students should be supported and developed starting at a very early age together with the development of other important competences including team work and presenting (orally and in written form) to various audiences. Some of the students' findings were on such a high level that they were published (and in some cases quoted by specialists) in mathematics research journals. The High School Student's Institute plays an important role with respect to the dissemination of IBL because the participating students (and their mentors) are coming from different towns in the country. Another positive effect is that a number of HSSI alumni act as mentors (virtually and face-to-face), thus passing the torch on to the next generation of young researchers.

4 Germany: Multipliers Concept for the Urban Network of Primary Schools

The University of Bayreuth, Germany, developed a practice-based multipliers concept for an urban network of primary schools with the aim to develop mathematics education and to support pupils' key competences. The idea is implemented on a local scale and addresses primary school teachers of the city Augsburg. The concept is realised within the framework of the European project KeyCoMath. 28 primary schools are taking part in a face-to-face professional development.

4.1 Involved Parties

On the urban scale, the University of Bayreuth coordinates, organises and supervises activities for the professional training of primary school teachers. It works directly with a group of qualified maths teachers as coaches for teachers' professional development. These teachers with their advisory function form a link between university and school and between science and practice, because they are responsible for appointed mathematical tutors from primary schools in the entire city.

4.2 Organisation

The Chair of Mathematics and Didactic of Mathematics is regularly in contact with the local education authority and with nine dedicated teachers who were assigned to become advisors due to their high professional expertise and their innovation capacity. Groups of two or three teacher-advisors guide 28 primary schools in the entire city. Therefore, a classification into four school groups according to location (north, south, east and west) has been assigned. Every primary school appoints at least two mathematical tutors – one responsible for the first and second form, the other for the third and fourth form. Hence, one school group consists of a minimum of twelve mathematical tutors who will further introduce/present newly gained acquaintances to their local colleagues.

4.3 Functions

The team at the University of Bayreuth has several duties: they keep in touch and make all arrangements with the local education authority. Teacher-advisors are appointed and meetings are summoned. They provide an academic support for schools and make funds as well as learning and teaching materials available. Above all, they operate a hosting platform for the information and material exchange. The nine mathematical teacher-advisors meet the university team regularly and they organise and guide school group meetings. There, they assist the mathematical tutors in planning teaching units. The tutors take an active part in the meetings in their assigned school group. They prepare and realise planned mathematical lessons and document teaching approaches.

4.4 Activities

Once a year, a major event for all involved primary school teachers takes place at the university. Such a meeting consists of a lecture to a superordinate topic like developing key competences by mathematics education. The attending teachers receive theoretical input and adopt it afterwards in workshops. The inclusion of pupils' utterances and solution processes conditioned by the study of pupils' written exercises, call logs or video shots demonstrates how children operate in a specific lesson's sequence.

In addition, every local primary school has the possibility to book in-house advanced training courses for its teaching staff according to the requirements. Normally, four school group meetings per school year are organised by the teacher-advisors. For these meetings, the mathematical tutors have to prepare the following: They previously realise a teaching unit in their own class and collect pupils' approaches. Above all, the members of a school group organise joint visits to classes, compare their varying approaches of teaching on the same specific mathematical topic, together reflect on one's own work and exchange their ready-made experiences, whereby mathematics education can be refined.

The focus is not solely set on spreading teaching materials and on giving recommendations for lessons. The objective of the project lies to a great degree on developing the level of inner convictions and beliefs teachers have of their subject, on teaching and learning processes, and their role in the lessons. These play a major role for the planning and implementation of lessons (Blömeke et al. 2010; Kunter et al. 2011).

4.5 Experience

This multipliers concept for the urban network of primary schools has proven itself over years and will be continued in future.
A coordinator says: *"It is encouraging to witness how mathematics education progresses at many primary schools in Augsburg, how teachers set out together to develop good mathematical assignments and how they bring teaching concepts of the common retrospect into question and thus improve them. By observing the children, I recognized their growing interest for mathematical issues, their creative dealing with numbers, patterns and structures, a greater openness for mathematical observations of everyday life and an increasingly safer handling with basic mathematical skills and abilities. The cooperation with school administration (education authority, administration) is successful because of good personal contacts."*

The involved primary teachers work as multipliers during their leisure time, including an additional effort in their daily workload, which is unfortunately neither compensated nor squared with class hour reduction, but which demonstrates the commitment of the participating teachers – since *"Staying still means taking a step back"* (a teacher advisor).

Concerning the observation of lessons, it is not always easy to find a volunteer willing to present the own teaching approach to colleagues. However, this risk is effectively prevented by the offer of a joint preparation of classes. *"I can work through exciting new ideas and discuss problems with motivated colleagues. Instead of sitting alone in my room I have many like-minded people around me."* (a participating teacher) and *"At the moment it's the only opportunity to get a direct look at the work of my colleagues. I can challenge myself with the implementation of the curriculum and at the same time receive inspiration through the teachers and their contact with students. Everyone learns from each other in a relaxed atmosphere without the pressure of being judged."* (a teacher who is on parental leave)

5 South Tyrol: Cooperative Design of Learning Environments

In the framework of KeyCoMath the German Department of Education in South Tyrol organised a teacher training for the development of competence-focused tasks for mathematics education at secondary schools. In a series of meetings the teachers were made acquainted with pedagogical and didactical theories concerning

- open questions,
- networking,
- individual learning,
- dialogic learning and
- assessment.

The participants, working in teams, developed tasks for their own lessons. These were tested in the classes and then, after a common reflection, optimised. The tasks focused on the acquisition of mathematical competences, but also on communicative, social and digital competences. The best practice examples of the tasks were published in a printed book „Tasks for competence-focused teaching of mathematics“ (Höller, Ulm 2015) and on the website www.KeyCoMath.eu.

Strategies for Teachers' Professional Development
Particularly valuable was the common work on tasks in the team of teachers – from the conception to the reflection and optimisation. Therefore, the planning of further training sessions will not only include contents and methods, but also ways of cooperative learning. The cooperation of the participants will take place as:

- teaching development in peer groups,
- common learning with mutual observation,
- exchanging views through e-learning platforms.

The focus lies on the development of teaching by the common design of learning environments for students. Moreover, the issue of evaluation is included.

These basic strategies for teachers' professional development are shown by the following diagram:

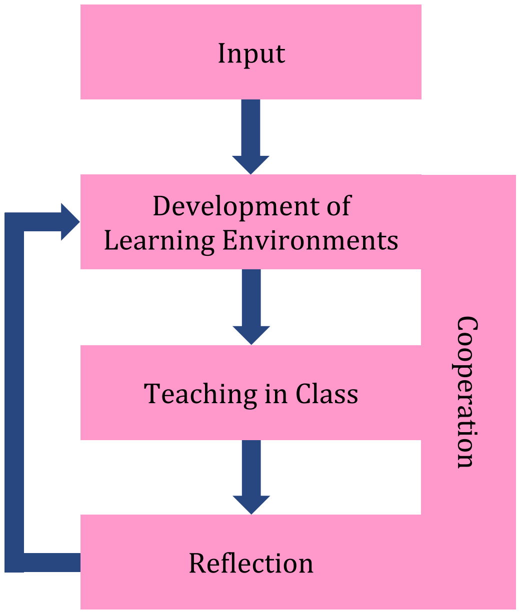

Fig. 5. Teachers' professional development by working with learning environments

6 Conclusion

In the project KeyCoMath partners from eight European countries developed concepts for innovating mathematics education. The focus lay on supporting students' key competences. Although the approaches in the different countries had to consider national frameworks, a common pattern of the strategies can be identified.

Aiming at Teachers

The key people for innovations in school are the teachers. Their beliefs, motivation and professional expertise are crucial for everyday teaching and learning. Thus, KeyCoMath focused on the teachers' professional development.

Networks of Schools

Since learning is a social process, regional networks of schools were established. They offered frameworks for teachers' exchange of experience and for their cooperative learning.

Coaches for Teachers' Professional Development

The regional school networks were led by a coach or a team of coaches who could be e.g. very experienced teachers, teacher educators or scientists. The teachers were made familiar with general didactical and pedagogical concepts. They related these ideas to their daily work at school; they designed learning environments for their students, and used them in their classes. The teachers presented, discussed and reflected their experiences cooperatively in their network of schools guided by their coach.

Aiming at the Meta-Level

Initiatives for substantial innovations of the educational system should aim at the meta-level of teachers' attitudes and beliefs. This concerns e.g. the role of the teacher, the role of the students, the nature of the subjects and general aims of education.

Development of Learning Environments

To bridge the gap between theory and practice, teachers individually and cooperatively developed learning environments for their students, worked with them in class, optimized them on the basis of all experiences, and exchanged and discussed them in their school network. Thus, by designing and working with specific learning environments teachers became acquainted with general pedagogical ideas. Learning environments function as tools for systemic teachers' professional development.

Areas of Activity

Participating schools and teachers should concentrate on one or a few areas of activity, e.g. exploratory learning, promoting students' cooperation, cumulative learning or fostering key competences. Such bounded fields of activity enable teachers to begin with substantial changes without the risk of losing their professional competence in class.

Universities as Innovation Centres

In these processes teachers and coaches received guidance and advice from universities. They served as innovation centres for teacher education. They provided regular and systematic in-service teacher education offers and coached the coaches.

(Inter-)National Teacher Education

Teachers and teacher educators were given the opportunity to exchange experiences with colleagues and to participate in professional development offers on a regional or a national level. They understood that problems and necessities of innovations have systemic character and concern the fundaments of education far beyond their own professional sphere. Moreover, they received ideas for innovation activities from a large community.

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